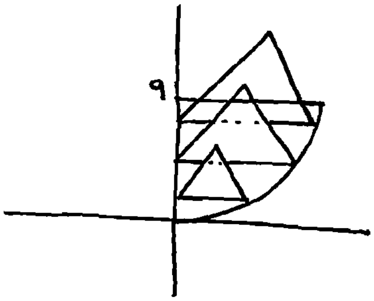
Volume Test review

①



base : $\sqrt{y}$

$A = \frac{\sqrt{3}}{4}\left(\sqrt{y}\right)^2$

$y = x^2$

$x = \sqrt{y}$

$V = \frac{\sqrt{3}}{4}\int_0^9 \left(\sqrt{y}\right)^2 dy$

$V = \frac{\sqrt{3}}{4}\left[\frac{1}{2}y^2 \Big|_0^9\right] = \boxed{17.537}$

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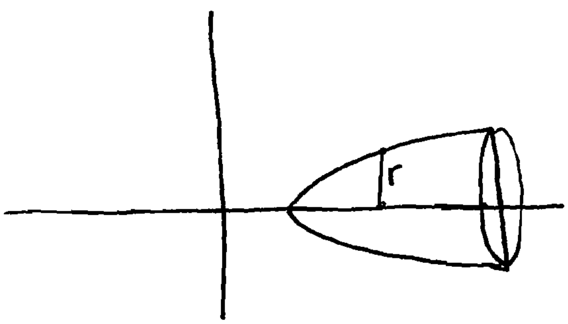
②



Bounds :

$\ln(x) = 0$

$x = 1$

Given

$x = 5$

$r(x) = \ln(x)$

$\pi \int_1^5 \left(\ln(x)\right)^2 dx = \boxed{4.857\pi}$

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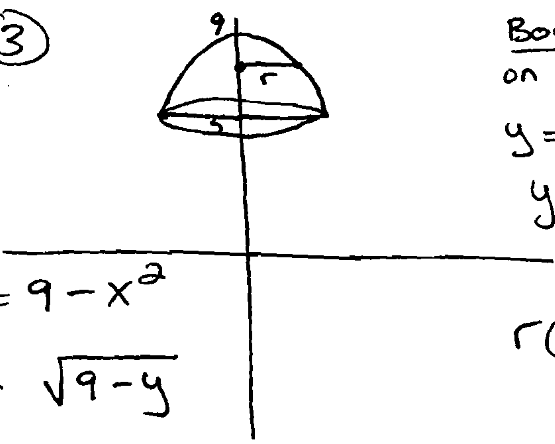
③



Bounds :

on y-axis, $x = 0$

$y = 9 - (0)^2$

$y = 9$

Given

$y = 5$

$y = 9 - x^2$

$x = \sqrt{9-y}$

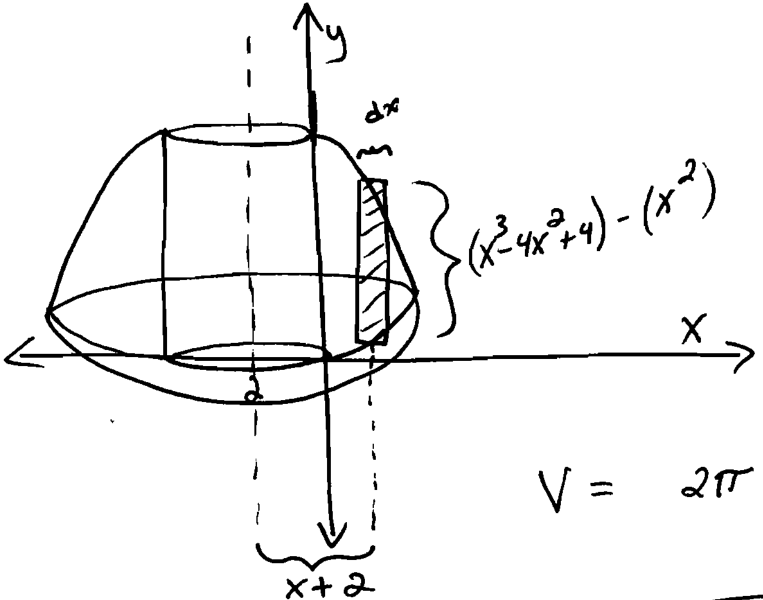
$r(y) = \sqrt{9-y}$

$\pi \int_5^9 \left(\sqrt{9-y}\right)^2 dy$

$\pi\left[9y - \frac{1}{2}y^2 \Big|_5^9\right] =$

$\boxed{8\pi}$

(4)



Bounds: [0, 1]

$h = x^3 - 5x^2 + 4$

$w = dx$

$p = x + 2$

$$V = 2\pi \int_0^1 (x^3 - 5x^2 + 4)(x+2)\, dx =$$

$\boxed{12.2333\,\pi}$

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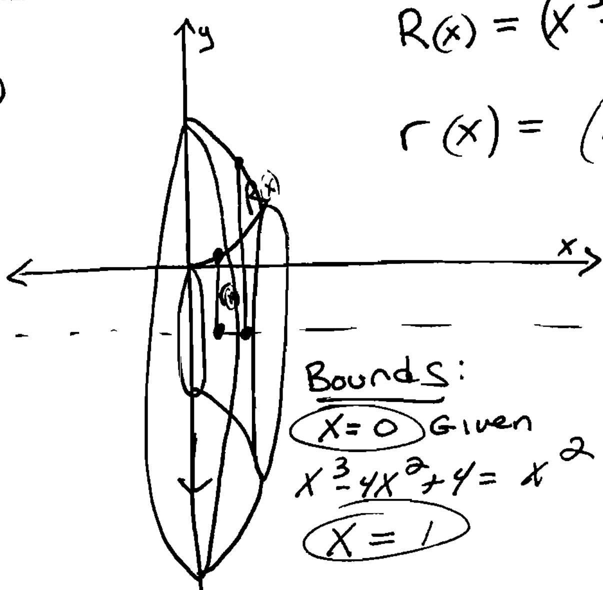
(5)



$R(x) = (x^3 - 4x^2 + 4) + 1 = x^3 - 4x^2 + 5$

$r(x) = (x^2) + 1 = x^2 + 1$

$$\pi \int_0^1 \left[(x^3 - 4x^2 + 5)^2 - (x^2+1)^2\right] dx$$

Bounds:

X = 0 Given

$x^3 - 4x^2 + 4 = x^2$

X = 1

$\boxed{14.3095\,\pi}$

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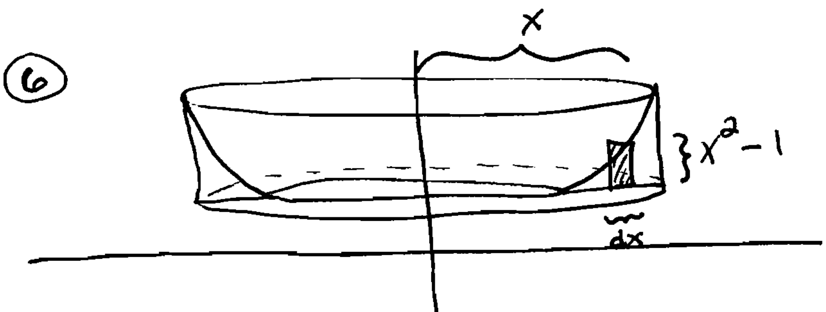
(6)



$h = x^2 - 1$ $w = dx$ $p = x$

$$2\pi \int_1^3 x(x^2 - 1)\, dx =$$

$\boxed{32\,\pi}$

Bounds (in terms of x):

Given: $\boxed{x = 3}$

$x^2 = 1$

$x = \pm 1$

Neg is not in area bound so $\boxed{x = 1}$

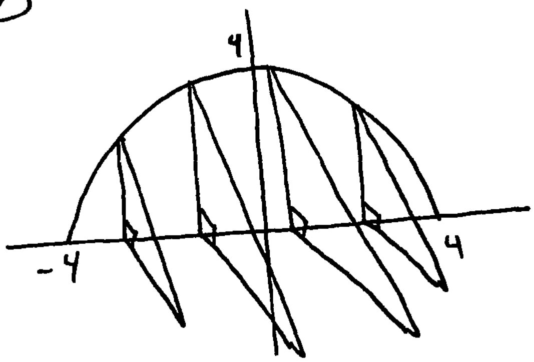
washer method:

$$\pi \int_1^9 \left[(3)^2 - (\sqrt{y})^2\right] dy =$$

$\boxed{32\,\pi}$

*Show how you get bounds and draw radii on picture.

(7)



$b = \sqrt{16 - x^2}$

$A = \frac{1}{2}\left(\sqrt{16-x^2}\right)^2$

$V = \frac{1}{2}\int_{-4}^{4} \left(\sqrt{16 - x^2}\right)^2 dx =$

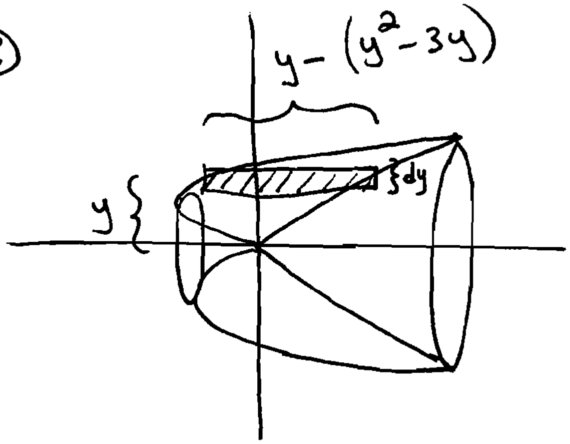
Bounds:

$\sqrt{16 - x^2} = 0$

$X = \pm 4$

$\boxed{42.6667}$

(8)



Bounds:

$y^2 - 3y = y$

$y^2 - 4y = 0$

$y(y-4) = 0$

$\boxed{y = 0 \quad y = 4}$

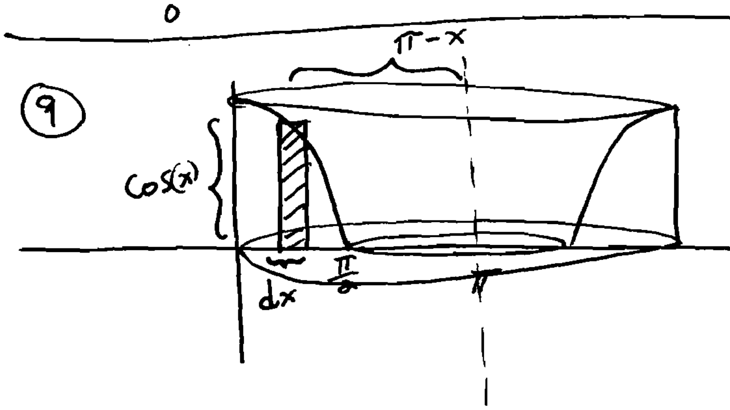
$h = -y^2 + 4y$

$w = dy$

$p = y$

$$2\pi \int_0^4 y(-y^2+4y)\, dy = \boxed{42.6667\pi}$$

(9)



$h = \cos(x) \quad w = dx \quad p = \pi - x$

$$2\pi \int_0^{\pi/2} \cos(x)(\pi - x)\, dx =$$

$\boxed{5.1416\,\pi}$

Bounds:

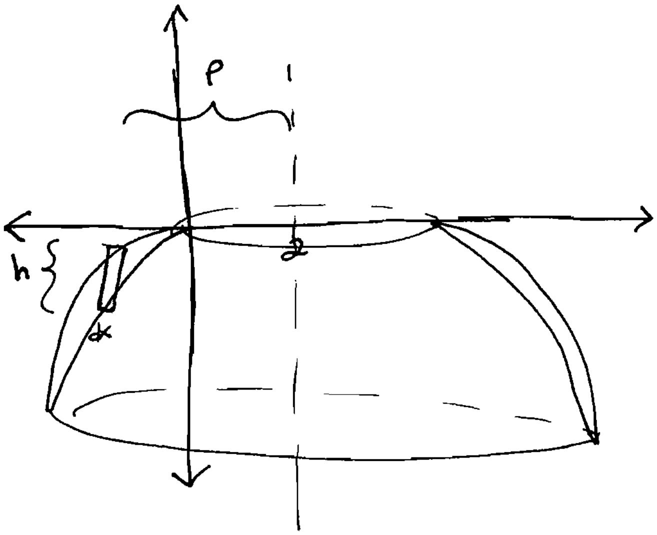
$\cos(x) = 0$

$\boxed{X = \frac{\pi}{2}}$

Given

$\boxed{X = 0}$

(10) $f(x) = x^3 \quad (-\infty, 0]$, $f(x) = -x^2 + 2x$



Bounds :

$x^3 = -x^2 + 2x$

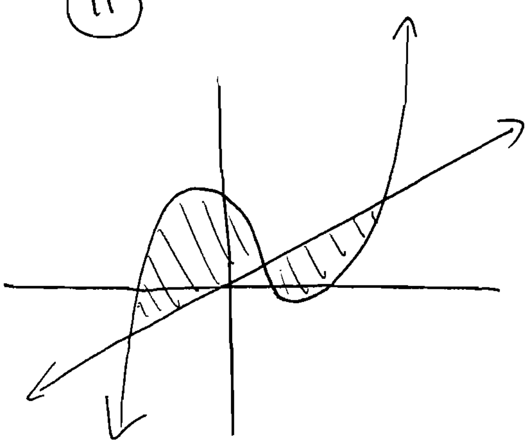
$x = -2, 0, \cancel{1}$

$x = 1$ is not in domain

$w = dx \qquad h = x^3 - (-x^2 + 2x) \qquad P = 2 - x$

$$V = 2\pi \int_{-2}^{0} (2-x)(x^3 + x^2 - 2x)\, dx = \boxed{51.941 \ u^3}$$

(11)



Bounds : $f(x) = g(x)$

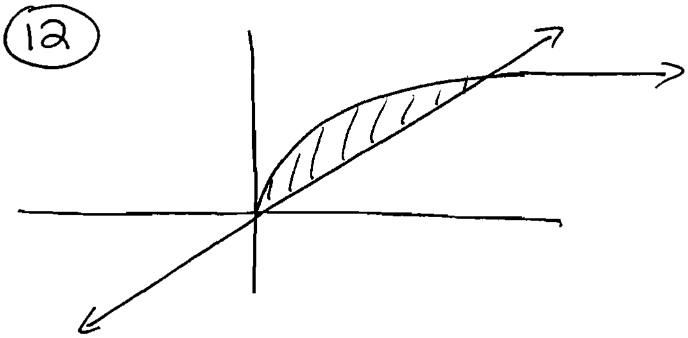
$x = -2.564, 0, 2.564$

$$A = \int_{-2.564}^{0} (f(x) - g(x))\, dx + \int_{0}^{2.564} (g(x) - f(x))\, dx$$

$8.529 + 8.529$

$\boxed{17.058 \ u^2}$

(12)



Bounds:

$2\sqrt{x} = \frac{1}{2}x$

$4\sqrt{x} = x$

$16x = x^2$

$x^2 - 16x = 0$

$x(x-16) = 0$

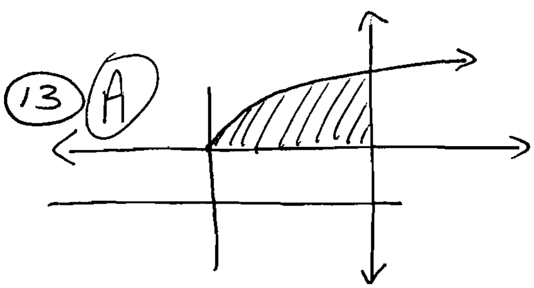
$x = 0, 16$

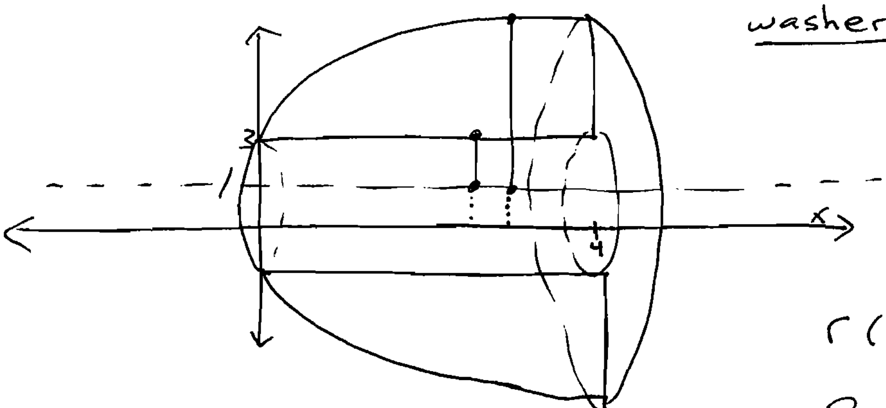
$$A = \int_0^{16} \left(2\sqrt{x} - \frac{1}{2}x\right) dx$$

$$\frac{4}{3}x^{\frac{3}{2}} - \frac{1}{4}x^2 \Big|_0^{16}$$

$$\left(\frac{256}{3} - 64\right) - (0) = \boxed{\frac{64}{3}\ u^2} \text{ or } \boxed{21.3333\ u^2}$$

(13) (A)





washer: Bounds:

$\sqrt{x} + 3 = 3$

$x = 0$

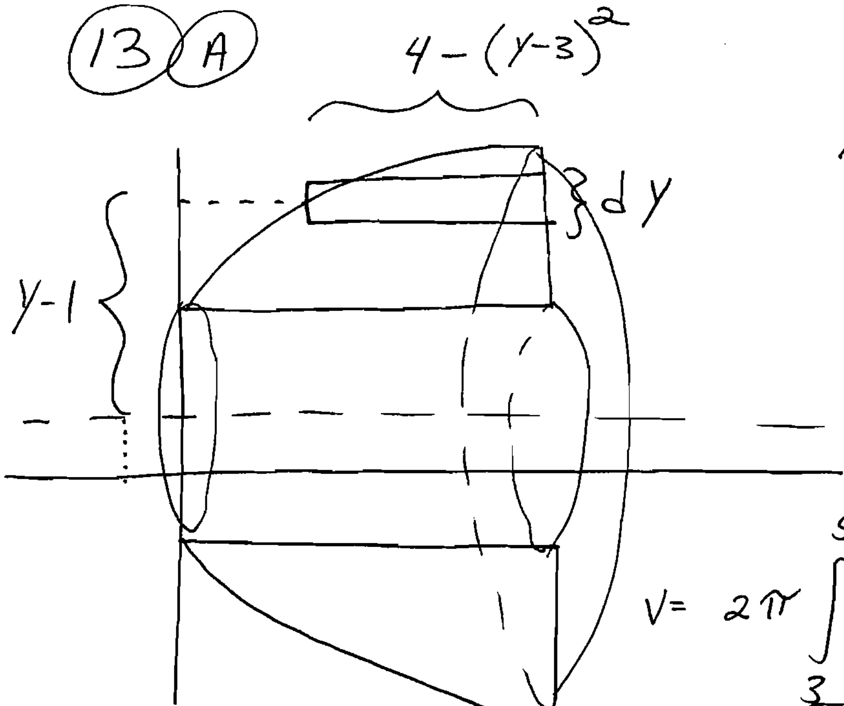
$x = 4$ given

$r(x) = 3 - 1 = 2$

$R(x) = (\sqrt{x} + 3) - 1 = \sqrt{x} + 2$

$$V = \pi \int_0^4 \left((\sqrt{x}+2)^2 - 2^2\right) dx = \boxed{92.1534\ u^3}$$

13 A



$y = \sqrt{x} + 3$

$x = (y-3)^2$

Bounds:

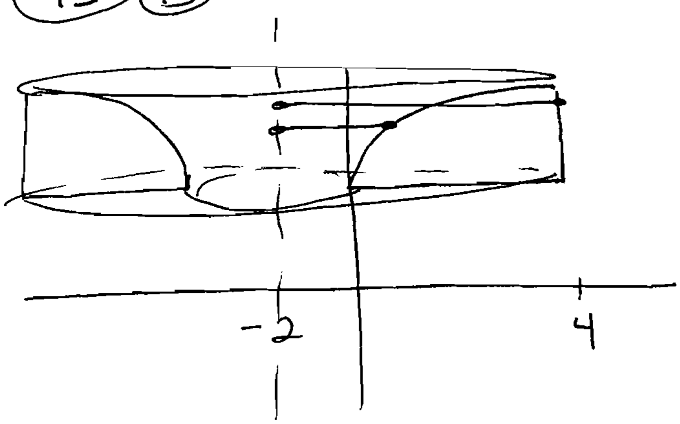
$y = 3$ given

$f(4) = 5 = y$

$$V = 2\pi \int_3^5 (y-1)\left[4-(y-3)^2\right] dy$$

$\boxed{92.1534\ u^3}$

13 B



washer:

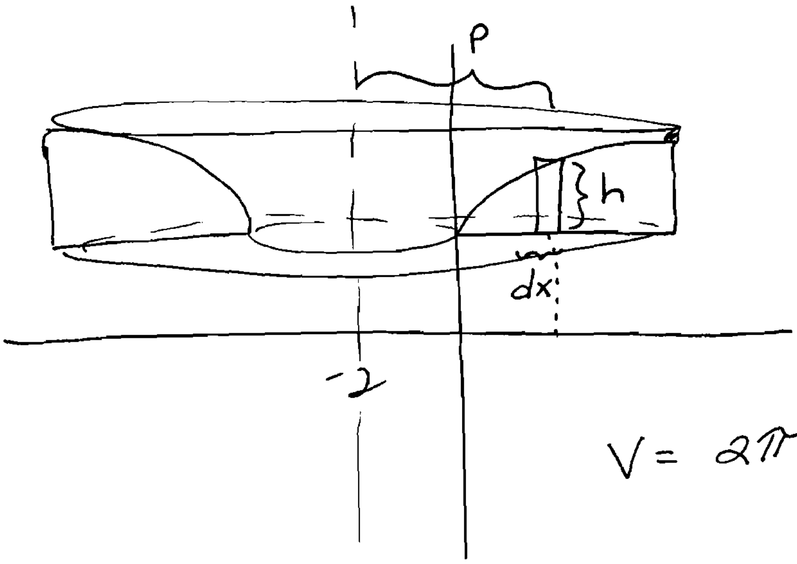
$R(y) = 4 - {}^{-}2 = 6$

$r(y) = (y-3)^2 - {}^{-}2 = (y-3)^2 + 2$

$$V = \pi \int_3^5 \left(6^2 - \left[(y-3)^2 + 2\right]^2\right) dy$$

$\boxed{147.4454\ u^3}$

13 B



Shell:

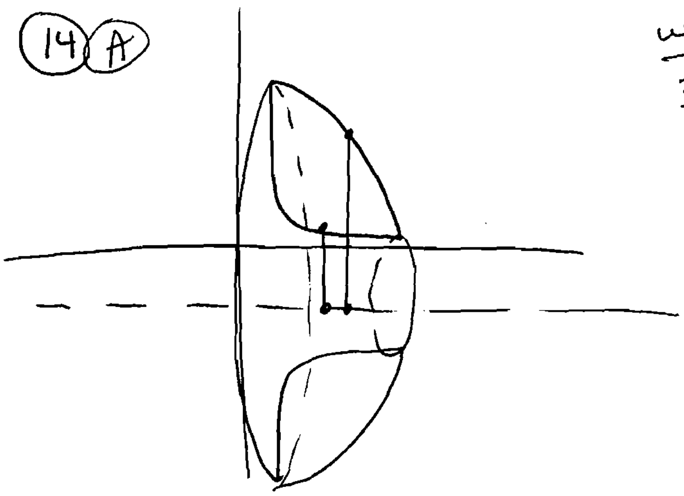
$h = (\sqrt{x} + 3) - 3 = \sqrt{x}$

$p = x - {}^{-}2 = x + 2$

$$V = 2\pi \int_0^4 \sqrt{x}\,(x+2)\,dx$$

$\boxed{147.4454\ u^3}$

14 A



washer:

Bounds: $f(x) = g(x)$

$x = .5083, 2.7824$

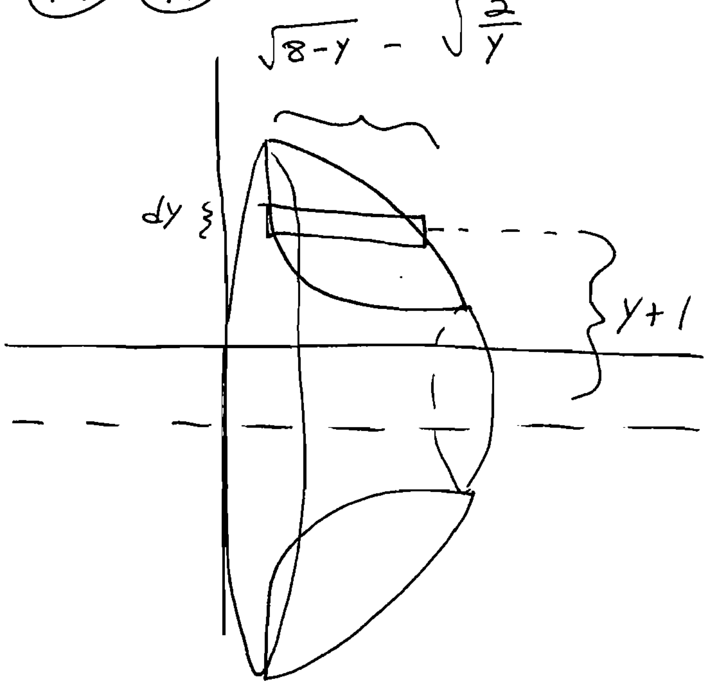
$r(x) = \frac{2}{x^2} - {}^{-}1 = \frac{2}{x^2} + 1$

$R(x) = -x^2 + 8 - {}^{-}1 = -x^2 + 9$

$$V = \pi \int_{.5083}^{2.7824} \left[(-x^2+9)^2 - \left(\frac{2}{x^2} + 1\right)^2\right] dx$$

$\boxed{220.8392\ u^3}$

14 A



Shell: $y = \frac{2}{x^2}$

$X = \sqrt{\frac{2}{y}}$

$y = -x^2 + 8$

$X = \sqrt{8-y}$

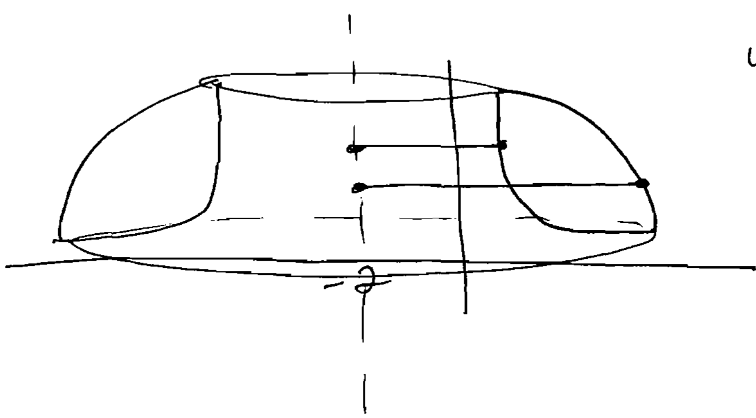
Bounds:

$\sqrt{\frac{2}{y}} = \sqrt{8-y}$

$y = 7.7417\ ,\ .2583$

$$2\pi \int_{.2583}^{7.7417} (y+1)\left(\sqrt{8-y} - \sqrt{\tfrac{2}{y}}\right) dy = \boxed{220.8392\ u^3}$$

14 B



washer:

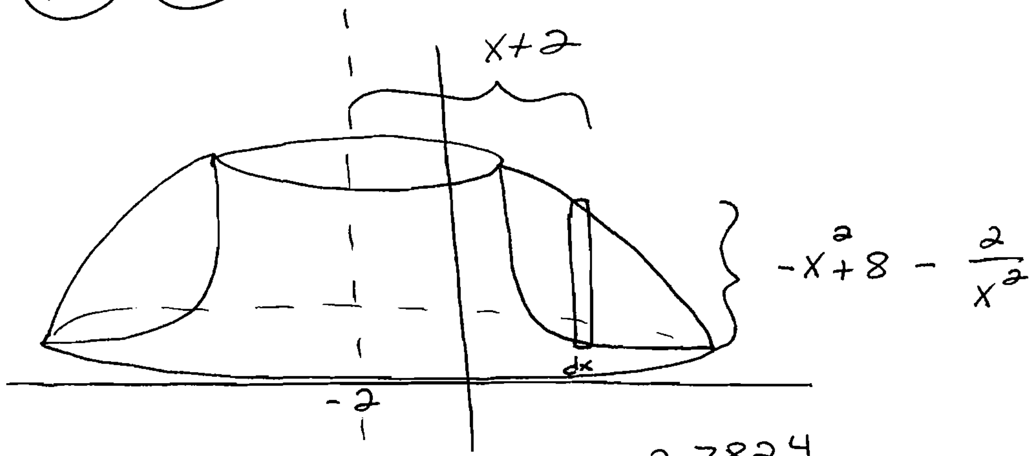
$r(y) = \sqrt{\frac{2}{y}} + 2$

$R(y) = \sqrt{8-y} + 2$

$$V = \pi \int_{.2583}^{7.7417} \left[\left(\sqrt{8-y} + 2\right)^2 - \left(\sqrt{\tfrac{2}{y}} + 2\right)^2 \right] dy$$

$$\boxed{171.2011\ u^3}$$

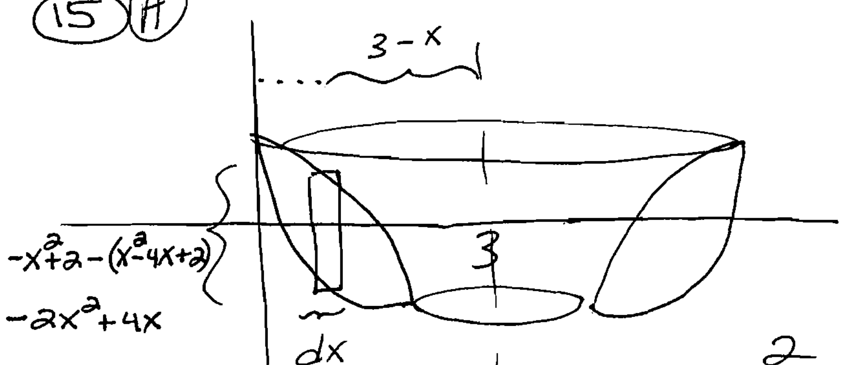
14 B



$$V = 2\pi \int_{.5083}^{2.7824} (x+2)\left(-x^2+8-\frac{2}{x^2}\right) dx$$

$$\boxed{171.2011 \ u^3}$$

15 A



<u>Bounds</u>:

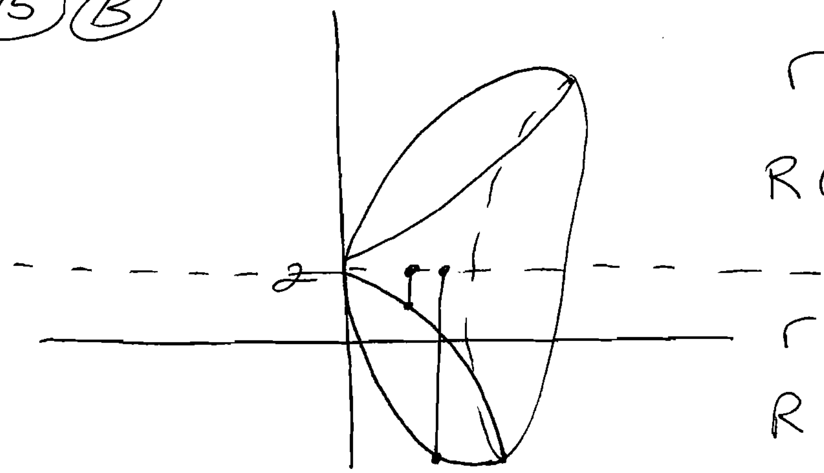
$f(x) = g(x)$

$x = 0, 2$

$$2\pi \int_{0}^{2} (3-x)(-2x^2+4x)\, dx$$

$$\boxed{33.5103 \ u^3}$$

15 B



$r(x) = 2-(-x^2+2)$

$R(x) = 2-(x^2-4x+2)$

$r(x) = x^2$

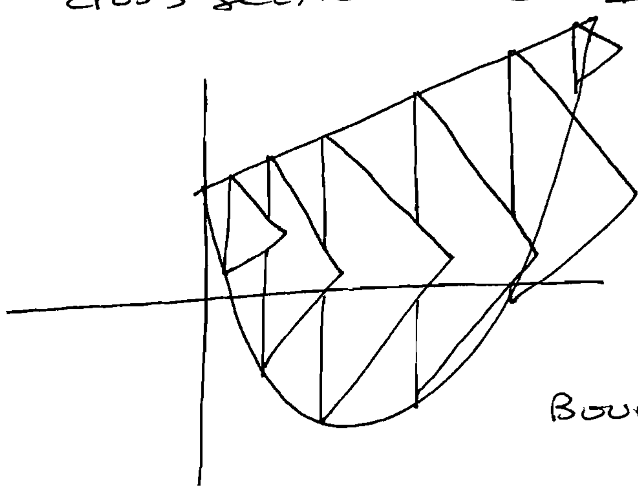
$R(x) = -x^2+4x$

$$\pi \int_0^2 \left(\left(-x^2+4x\right)^2 - \left(x^2\right)^2\right) dx = \boxed{33.5103 u^3}$$

Extra cross section Problem:

Area bound by $f(x) = x^2-5x+2$; $g(x) = x+2$

cross sections are $\perp$ to x-axis and are equilateral Δs.



Base of one $\Delta = x+2-(x^2-5x+2)$

$= -x^2+6x$

Area of 1 $\Delta = \frac{\sqrt{3}}{4}\left(-x^2+6x\right)^2$

Bounds: $f(x) = g(x)$

$x = 0, 6$

$$V = \frac{\sqrt{3}}{4}\int_0^6 \left(-x^2+6x\right)^2 dx =$$

$\boxed{15.5885 u^3}$